Advanced Topics in Random Graphs
Exercise Sheet 1

Question 1. Show that for most matrices A any linear program which computes the transformation $x \mapsto Ax$ requires $\Omega\left(\frac{n^2}{\log n}\right)$ gates.

(*) Show that the underlying graph of any linear program which computes the transformation $x \mapsto Ax$ for a super regular matrix A must be a super concentrator.

Question 2. Show using a greedy argument that there exists an $\epsilon > 0$ such that there exist arbitrarily large dictionaries with rate and distance at least ϵ.

Question 3. Verify that there is some constant n_0 such that if $d \geq 32$, $n \geq n_0$, $m \geq \frac{3n}{4}$ and $\frac{n}{10d} < s \leq \frac{n}{2}$

$$\left(\frac{en}{s}\right)^s \left(\frac{em}{s}\right)^s \left(\frac{s}{m}\right)^{sd} < 20^{-s},$$

and if $s \leq \frac{n}{10d}$ then

$$\left(\frac{en}{s}\right)^s \left(\frac{8em}{5ds}\right)^{\frac{5ds}{8}} \left(\frac{5ds}{8m}\right)^{sd} \leq 20^{-s}.$$

Question 4. Let G be an (n, d)-graph and let $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_n$ be the eigenvalues of G with associated eigenvectors $v_1, v_2, \ldots, v_n$. Show that

i) $\lambda_1 = d$ and the corresponding eigenvector is $v_1 = \frac{1}{\sqrt{d}}\mathbf{1} = (\frac{1}{\sqrt{d}}, \frac{1}{\sqrt{d}}, \ldots, \frac{1}{\sqrt{d}})$.

ii) G is connected iff $\lambda_1 > \lambda_2$.

iii) A connected graph G is bipartite iff $\lambda_1 = -\lambda_n$.

Question 5. Let G be an (n, d, α)-graph. Show that

i) The largest independent set in G has size at most αn.

ii) G has chromatic number at least $\frac{1}{\alpha}$.

iii) G has diameter $O(\log n)$.